

Name: \_\_\_\_\_

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**Math 10/11 Enriched: Section 5.1 Non Permissible Values & Simplifying Rational Expressions:**

1. What is a Non-Permissible Value?. How do you find NPV's in a rational expression? Explain:

2. The expression below  $r_1, r_2, \dots, r_n$  are all positive integers. What are all the NPV's?

i)  $\frac{(x-r_1)(x+r_2)}{x^2-r_3} \div \frac{x-r_4}{2x+r_5}$

ii)  $\frac{1}{x} + \frac{1}{x+r_4} - \frac{x}{x^3-r_5}$

3. Find the non-permissible value(s) for each of the following expressions:

|                                                           |                                                                      |                                                                                    |
|-----------------------------------------------------------|----------------------------------------------------------------------|------------------------------------------------------------------------------------|
| a) $\frac{2x}{3(x-1)(3x+4)}$<br><br>NPV:                  | b) $\frac{3x^3-2}{x^2-6x-16}$<br><br>NPV:                            | c) $\frac{x+3}{x^3-27}$<br><br>NPV:                                                |
| d) $\frac{3x+10}{x^3+10x}$<br><br>NPV:                    | e) $\frac{3+2x}{\sqrt{3x+12}}$<br><br>NPV:                           | f) $\frac{3x}{2} - \frac{6}{2x^2-1}$<br><br>NPV:                                   |
| g) $\frac{12x^3}{2^{2x}-16}$<br><br>NPV:                  | h) $\frac{8x^3-2x}{5x(x^4-13x^2+36)}$<br><br>NPV:                    | i) $\frac{\frac{2}{2x}-3}{9-\frac{x}{3}}$<br><br>NPV:                              |
| j) $\frac{2x-3}{x+2} \div \frac{3x+4}{x^2-9}$<br><br>NPV: | k) $\frac{x+14}{x^2-16} \div \frac{x^2-5x+14}{x^2-2x+8}$<br><br>NPV: | l) $\frac{\frac{2}{x} - \frac{3}{x+2}}{\frac{1}{x-2} - \frac{x}{x-1}}$<br><br>NPV: |

4. Add or Subtract each of the following expressions below. State all the NPV's

|                                                          |                                                           |
|----------------------------------------------------------|-----------------------------------------------------------|
| a) $\frac{2}{x+5} + \frac{3}{x+2}$                       | b) $\frac{x-2}{x+2} + \frac{x+1}{x-4}$                    |
| c) $\frac{3}{x-1} - \frac{2}{2-x}$                       | d) $\frac{5x}{3x+9} - \frac{9x}{2x+6}$                    |
| e) $\frac{2}{x-2} + \frac{2x}{2-x} + \frac{x+2}{x-2}$    | f) $\frac{4x^2-20x}{x^2+2x-35} + \frac{3x-6}{x^2-12x+20}$ |
| g) $\frac{a}{a+b} - \frac{b}{b-a} + \frac{2ab}{a^2-b^2}$ | h) $\frac{2x-6}{x^2-5x+6} - \frac{3x-12}{x^2-x-12}$       |

|                                                                 |                                                              |
|-----------------------------------------------------------------|--------------------------------------------------------------|
| i) $\frac{2x}{3x^2 - 11x + 6} - \frac{3x - 12}{3x^2 - 14x + 8}$ | j) $\frac{2x}{3 - x} - \frac{3x}{x + 3} + \frac{2}{x^2 - 9}$ |
|-----------------------------------------------------------------|--------------------------------------------------------------|

5. Multiply or Divide each of the following expressions below. State all the NPV's

|                                                                      |                                                                             |
|----------------------------------------------------------------------|-----------------------------------------------------------------------------|
| a) $\frac{(2x)^2}{5y} \times \frac{10x}{8y} \div \frac{15x}{(4y)^2}$ | b) $\frac{15x}{2x + 6} \div \frac{10x}{3x + 9}$                             |
| c) $\frac{(x+1)^2}{x^2 - 1} \times \frac{x^2 - 4}{x^2 + 3x + 2}$     | d) $\frac{x^2 + 5x + 6}{x^2 - 5x + 6} \div \frac{x^2 - x - 6}{x^2 + x - 6}$ |
| e) $\frac{x^2 - 16y^2}{6x^2y} \div \frac{x^2 + xy - 20y^2}{4x^3y^2}$ | f) $\frac{x^2 + 4x - 5}{3x - 6} \times \frac{x - 2}{1 - x} =$               |

|                                                                                         |                                                                                            |
|-----------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|
| g) $\frac{m^2 - 9mn + 14n^2}{m^2 + 7mn + 12n^2} \div \frac{3m^2 - 21mn}{4m^3 + 16m^2n}$ | h) $\frac{3x^2 + 3x - 6}{x^2y - 7xy} \times \frac{x^2y - 13xy + 42y}{6x^2 + 12x}$          |
| i) $\frac{x+2y}{x-3y} \times \frac{x^2-9y^2}{x^2-4y^2} \div \frac{x+3y}{x-2y}$          | j) $\frac{(3a+7b)^2}{2a-5b} \times \frac{4a^2-25b^2}{9a^2-49b^2} \div \frac{2a+5b}{3a-7b}$ |

6. Simplify and find all the NPV's: i)  $\frac{\frac{x-2}{3} + \frac{1}{x^2+2x}}{\frac{2x}{6} - \frac{x+2}{6}}$

ii)  $\frac{y + \frac{2y}{y+2}}{1 + \frac{4}{y^2-4}}$

7. How many ordered pairs (a,b), where "a" and "b" are positive integers and a>b, satisfy the equation:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{32}?$$

8. What are all the integer values of “x” for which all three of the expressions below have integral values?

$$\frac{x}{x-2}, \frac{x}{x-4}, \text{ and } \frac{x}{x-6}$$

9. Given that  $\frac{1}{a(b+1)} + \frac{1}{b(a+1)} = \frac{1}{(a+1)(b+1)}$ , what is the value of  $\frac{1}{a} + \frac{1}{b}$ ?

10. Suppose that “c” and “d” are integers with  $c > 0$  and  $d > 0$ . If  $\frac{2c+1}{2d+1} = \frac{1}{17}$ , what is the smallest possible value of “d”? [Euclid]

11. Given the formula  $a_n = 1 + \frac{1}{1+a_{n-1}}$ , if  $a_3 = \frac{41}{29}$ , what is the value of  $a_1$  = ??

Warmup: Find the positive solution to the equation below: [Brilliant level 3]

$$\frac{1}{x^2 - 10x - 29} + \frac{1}{x^2 - 10x - 45} - \frac{2}{x^2 - 10x - 69} = 0$$

12.

13. Given that  $\frac{a}{b}$  is a fraction in lowest terms that satisfies the inequality:  $\frac{5}{9} < \frac{a}{b} < \frac{7}{12}$ . If the value of "b" is a positive value less than 50, what are all the possible fractions of  $\frac{a}{b}$ ?

14. For positive integers "a" and "b", define  $f(a,b) = \frac{a}{b} + \frac{b}{a} + \frac{1}{ab}$

- i) Determine the value of  $f(5,6)$
  - ii) Determine all the positive integers 'a' for which  $f(a,a)$  is an integer
  - iii) If "a" and "b" are positive integers and  $f(a,b)$  is an integer, prove that  $f(a,b)$  must be a multiple of 3
  - iv) Determine the four pairs of positive integers  $(a,b)$ , with  $2 < a < b$  for which  $f(a,b)$  is an integer.
- [Euclid]